MATHEMATICS WITHOUT BORDERS

AGE GROUP 6

SPRING 2020

INSTRUCTIONS

1. Please **DO NOT OPEN** the contest papers until the Exams Officer has given permission.

2. There are 20 questions with an open answer in the test.

3. Please write your answers in the ANSWER SHEET.

4. Each correctly solved problem earns 2 points, a partial solution earns 1 point, and unanswered or wrong answer gets 0 points.

5. The use of calculators or other electronic devices, as well as books containing formulae is NOT allowed during the course of the contest.

6. Working time: not more than 60 minutes. In the case of an equal number of solved problems, the higher ranked participant will be the one who has spent less time solving the problems.

7. No contest papers and draft notes can be taken out by any contestant.

8. Students are NOT allowed to receive help by the Exams Officer or by anyone else during the contest.

WE WISH YOU ALL SUCCESS!

Problem 1. Calculate

$$1 - 3 + 5 - 7 + 9 - 11 + \cdots + 21 - 23.$$

Problem 2. Calculate

$$(22 - 2) \times (20 - 4) \times (18 - 6) \times \ldots \times (4 - 20) \times (2 - 22).$$

Problem 3. Calculate the value of the following expression:

$$A = -1^5 + (-1)^5 - 1^4 + (-1)^4 - 1^3 + (-1)^3 - (-20) \times (-20).$$

Problem 4. How many natural numbers are there that are smaller than 12 and co-primes with 12.

Problem 5. Find the greatest negative integer that has 42 as the sum of its digits.

Problem 6. For how many primes N does the number equal to 3^N divide the product of all natural numbers from 1 to 20 with a remainder of 0?

Product 7. Find the greatest integer that is not greater than the following number.

$$\frac{5}{6} - \frac{7}{12} + \frac{1}{20} + \frac{11}{30}$$

Hint:

$$\frac{x \pm y}{x \times y} = \frac{1}{x} \pm \frac{1}{y}.$$

Problem 8. Find x if $4 \times 10^4 + x \times 10^2 + 5 \times 10^{-1} + 9 \times 10^{-4} = 40200.5009$.

Problem 9. Calculate

$$9 - x + |x - 9| + 10 - x + |x - 10|,$$

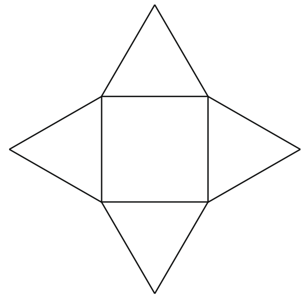
if $\boldsymbol{x} = 9.\overline{9}$.

Problem 10. The number

$$\frac{1}{12800}$$

is expressed as a decimal fraction. Find the number of digits after the decimal point.

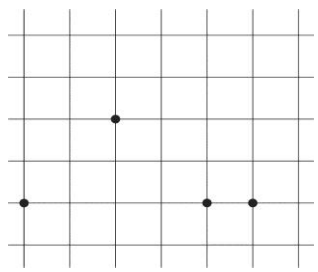
Problem 11. The figure is made up of a square and four equilateral triangles. Find the area of the square in square centimetres if the perimeter of the figure is 20 cm.



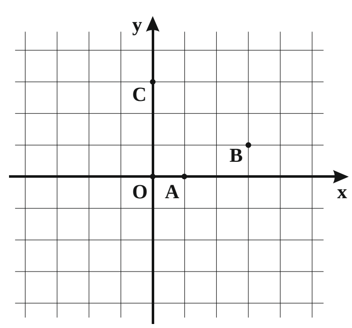
Problem 12. A rectangle is divided into three identical squares. Each square has an area of 361 cm^2. Find the perimeter of the rectangle.

Problem 13. A prism has 13 faces. How many vertices does it have?

Problem 14. 4 points have been marked on the square grid below. Three of them have the following coordinates: (0; 2), (0; -2) and (0; -3). Find the x-coordinate of the fourth point.



Problem 15. Calculate the area of the triangle *ABC*, if the coordinates of its vertices are *A* (1,0), *B* (3,1) and *C* (0,3), respectively.



Problem 16. What is the smallest prime that is a factor of the number equal to

$$1^2 + 3^3 + 5^5 + 7^7?$$

Problem 17. Calculate

$$\left|\frac{a}{|a|}+\frac{b}{|b|}+\frac{c}{|c|}\right|,$$

if

$$\frac{a}{|a|}+\frac{b}{|b|}+\frac{c}{|c|}<-1.$$

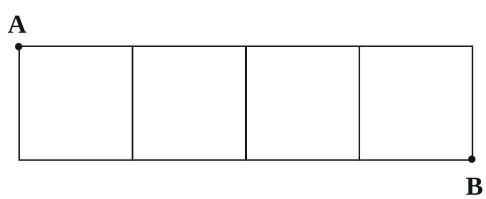
Problem 18. The sum of the digits of the two three-digit numbers *a* and *b* is 30. Find the least possible sum of the digits of the number equal to *a* + *b*.

Problem 19. Calculate

$$7\times\left|\frac{22}{7}-\pi\right|+7\pi\ +3.14.$$

$$\pi\approx 3{,}14159\ 26535\ 89793\ 23846\ 26433\ 83279\ 5$$

Problem 20. A rectangle has been built using 13 equal line segments with a total length of 91 cm:



How many routes with a length of 49 cm lead from point *A* to point *B*, if none of the line segments is used more than once?